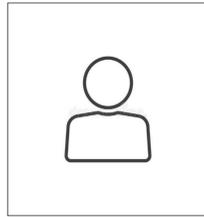


## APPROXIMATION OF MODELLING OF THE SHAPE OF THE FREE-HANGING STEEL ROPE ON THE EXAMPLE OF THE "POLINKA" GONDOLA CABLE CAR IN WROCLAW



Robert GRADKA<sup>1</sup>



Wojciech PALEJ<sup>1</sup>



Andrzej KWINTA<sup>2</sup>

<sup>1</sup>Wrocław University of Science and Technology, Faculty of Geoengineering, Mining and Geology, Department of Geodesy and Geoinformatics, Wybrzeże Wyspiańskiego 27, 50-370 Wrocław  
<sup>2</sup> University of Agriculture in Kraków, Faculty of Environmental Engineering and Land Surveying, Department of Geodesy, ul. Balicka 253a, 30-147 Kraków

### INTRODUCTION

In modern cities, gondola cable car are alternative means of public transport. The object which has been subjected to tests was a gondola cable car “Polinka” in the city Wrocław. The safety of use of this type of facilities requires periodic technical condition checks, including geometric elements of lines, supports and gondolas. As a result of geodetic measurements, it is possible to obtain data about the geometry of the lifting lines through the measured control points. The measurement can be performed using the tachimetric method based on the assumed measurement network in the area of the object. On the basis of the measured points, the equations of the theoretical curve (approximation) are determined. A parabola or a catenary is assumed as the theoretical curve. Then, the geometric parameters of the ropes are determined, which can be used during the assessment of the technical condition of the cable car. The calculation of the geometrical parameters of the ropes can be performed with the accuracy analysis.

### RESEARCH AREA

The measurement object was gondola cable car “Polinka” in the city of Wrocław. It has been in operations since 2013, over the river Oder, connecting the main campus of the Wrocław University of Technology and the building of Geocentrum, located on the other side of the river. This is a type of aerial cable car, construed of two carriages, suspended on two carrying-hauling lines. The cabins move in the opposite directions with a speed of up to 5 m.s<sup>-1</sup> and they are connected through a drive cable. Whilst, the pull cable is immobile. Each wagon may accommodate up to the maximum of 15 persons and the maximum capacity amounts to 366 persons per hour in both directions



Fig. 1. „Polinka” gondola cable car in Wrocław  
<https://www.radiowroclaw.pl/artykuly/widok/32180/Polinka-od-lutego-za-3-tyl-ROZKLAD>

### MATERIALS AND METHODS

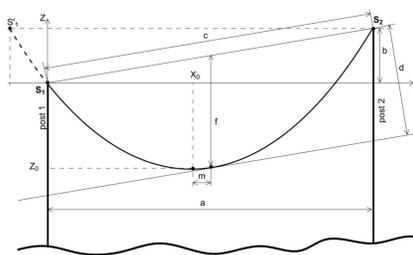


Fig. 2. Geometry of the single span

Equation of the free-hanging the span: catenary

$$Z(X) = k \cdot \left[ \cosh\left(\frac{X-u}{k}\right) - 1 \right] + w$$

parabola

$$Z(X) = pX^2 + qX + r$$

According to the figure 2 following geometric parameters can be distinguished in span:

- span length (a)** – a horizontal distance between axes of adjacent supporting structures (post1, post2)
- chord (c)** – a straight line segment, whose endpoints are cable support points
- sag (f)** – the biggest vertical distance between a cable and a chord
- slope (b)** – a vertical distance between two support points of one cable of an inclined span
- eccentricity (m)** – of an inclined span; a distance between the lowest point of a cable and a centre of a span
- deviation (d)** – the biggest distance between a curve of hanging cable and a chord; measured perpendicularly to a chord
- length (a)** – a length of a cable between two support points
- the lowest point (X<sub>0</sub>, Z<sub>0</sub>)** – coordinates of the lowest point of a cable

The tachimetric method combined with the GNSS method was chosen for the measurement of the main span. Multifunctional control points were established on the boulevard of the Wrocław University of Technology. Reference points are distributed as evenly as possible around the site. The measurement was performed using the fast static method with the use of two Trimble R6 receivers. The calculation of the network was carried out in the GNSS Solutions program. The tachimetric measurement of the lifting ropes was performed with a Trimble S3 total station.

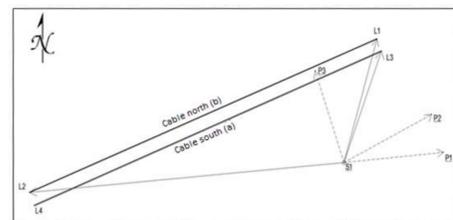


Fig. 3. The measurement grid and two measured cable line

Two lifting ropes, called the north and south ropes, were measured (Fig. 3). 30 points were measured for each of the ropes. Then the coordinates of the rope points were determined. The average error in the position of a point on the line in the horizontal plane was ± 0.01 m, and in the vertical plane it was ± 0.005 m. In the next step, the transformation to the local system was performed. The system was adopted so that the X axis passes through the points on the supports (in the rope plane).

### RESULTS

The calculation of the theoretical equations was carried out in accordance with the least squares method

$$\min_{k,u,w} \{Z^P, Z_C^T\} = \min_{k,u,w} \sum_{i=1}^n \left\| Z^P(X_i) - \left( k \cdot \left( \cosh\left(\frac{X_i-u}{k}\right) - 1 \right) + w \right) \right\|$$

Parabola

$$\min_{p,q,r} \{Z^P, Z_P^T\} = \min_{p,q,r} \sum_{i=1}^n \left\| Z^P(X_i) - (pX_i^2 + qX_i + r) \right\|$$

Tab. 1. Parameters of theoretical curves

Curve	Catenary	Parabola
Parameters	k= 2912.597±3.656 u= 152.848±0.049 w= 131.861±0.002	p= (171.7±0.2)×10 <sup>-6</sup> q=( -52.49±0.06)×10 <sup>-3</sup> r= 135.872±0.004
Residual sum of squares [VV]	0.0011348	0.001145
Mean error m <sub>0</sub> [m]	0.0065	0.0065
Correlation coefficient R	0.99998	0.99998

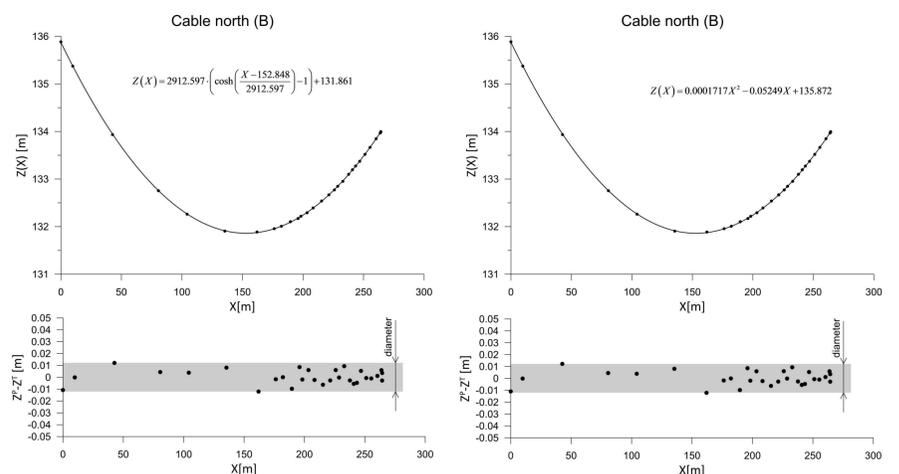


Fig. 4. Measured points and theoretical curves (top), difference between theoretical and measurement Z coordinate (bottom)

Tab. 2. Calculated geometry parameters of span for north cable

Parameter	Catenary	Parabola	Catenary	Parabola
a	a = max(X) – min(X)		264.360±0.014	
b	b =  Z(X = max(X)) – Z(X = min(X))		1.891±0.011	
c	c = √(a <sup>2</sup> + b <sup>2</sup> )		264.367±0.014	
X <sub>0</sub> , Z <sub>0</sub>	X <sub>0</sub> = u Z <sub>0</sub> = w	X <sub>0</sub> = - $\frac{q}{2p}$ Z <sub>0</sub> = $\frac{4pr - q^2}{4p}$	152.848±0.049 131.861±0.002	152.850±0.268 131.861±0.012
f	$f = \frac{b}{a} X_f + Z(X_{min}) - k \left[ \cosh\left(\frac{b}{a} \operatorname{arcsinh}\left(\frac{b}{a}\right)\right) - 1 \right] - w$ $X_f = k \cdot \operatorname{arcsinh}\left(\frac{b}{a}\right) + u$	$f = pX_f^2 + Z(X_{min}) - r$ $X_f = \frac{b - aq}{2ap}$	5.190±0.009 173.682±0.133	5.190±0.023 173.680±0.312
m	m = X <sub>f</sub> – X <sub>0</sub>		20.834±0.142	20.830±0.411
d	d = $\frac{a \cdot f}{c}$		5.190±0.009	5.190±0.023
α	$\alpha = k \left[ \sinh\left(\frac{X-u}{k}\right) \right]_{X_{min}}^{X_{max}}$	$\alpha = \frac{1}{4p} \left[ t\sqrt{t^2+1} + \operatorname{arcsinh}(t) \right]_{-2pX_{min}+q}^{2pX_{max}+q}$	264.457±0.014	264.457±0.541

### DISCUSSION AND CONCLUSIONS

- Total station measurements combined with GNSS measurements allow to obtain data for the analysis of the ropes of the gondola span. In the analyzed example, 30 points were determined for each rope.
- It is important to correctly measure the rope support points for measurements. This has an impact on the accuracy of determining the geometric elements of the span. Support points are not clearly determinable.
- A catenary or a parabola can be used to describe the theoretical geometry of a rope on a single span. The results of determining the geometric parameters for the northern rope are presented in Table 1 and Figure 4. It can be noticed that both curves describe the set of points well.
- Based on the theoretical curves, it is possible to calculate the geometrical parameters of the span. Table 2 presents the formulas for calculating the parameters of the span geometry and the determined parameter values along with the accuracy analysis. In a few cases, the obtained parameter values slightly differ (it is negligible).
- Some of the geometric parameters of the span are determined from the same data for both theoretical curves.
- The analysis of the accuracy of the obtained parameters of the span geometry shows that smaller errors of the span parameters are obtained on the basis of calculations for the catenary than for the parabola.
- In the case of the rope length, the error from the parabola is very large (almost 40 times) greater than the error from the catenary.